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Magnetoacoustic Waves in a Dissipative Quantum Plasma With Spin-1/2 and Exchange-Correlation Effects

Ibrahim G. H. Loqman[1,a)], Mahmood A. H. Khaled[1,2,b)] and Kauther I. Alkuhlani[1,c)]
[1]Department of Physics, Faculty of Science, Sana'a University, Sana'a, Yemen
[2]Department of Physics, Faculty of Education, Al-Mahweet, Sana'a University, Sana'a, Yemen

Abstract
Using quantum magneto-hydrodynamic theory, the nonlinear propagation of magnetoacoustic waves have been investigated in a quantum magneto-plasma having dissipative ions fluid as well as quantum electrons and positrons, including exchange-correlation and electrons/positrons spin effects. The Korteweg-devris-Burger equation is derived employing the reductive perturbation method. It has been found that the quantum magneto-plasma system under consideration supports buth magnetoacoustic solitary and shock waves depending on the values of the plasma parameters. The effects of quantum plasma parameters (such as exchange-correlation coefficients, magnetic field strength, kinematic viscosity and the concentrations of electrons and positrons) on the magnetoacoustic shock waves (both monotonic shock waves and oscillatory shock waves) are examined. The profiles of both monotonic and oscillatory shock waves are found to be significantly affected by these parameters.The results of the current study may be useful to understand the properties of magnetoacoustic waves propagating in dense space plasma environments where the quantum mechanical effects of electrons and positrons are included.

Keywords: Magnetoacoustic waves, Quantum plasma, Spin magnetization

الملخص: باستخدام النظرية الهيدروديناميكية المغناطيسية الكمية، تم دراسة الانتشار اللاخطي للموجات الصوتية المغناطيسية في بلازما كمية مكونة من الكترونات، بوزترونات كمية وايونات متدفقة في وجود تأثيرات اللف المغزلي و ارتباط التبادل لكلا الالكترونات والبوزترونات. تم اشتقاق معادلة كورتيفكي– دي فرسي برجر(Korteweg–de Vries–Burger equation) باستخدام نظرية الاضطراب المختزلة. وجدنا أن كل من الموجات السوليتونية والصدمية المرافقة للموجات الصوتية المغناطيسية يمكن أن توجد في نظام البلازما الكمية الحالي اعتماداً على قيم برامترات البلازما.تم دراسة تأثير بارامترات البلازما الكمية (مثل الترابط التبادلي للجسيمات الكمية، شدة المجال المغناطيسي، لزوجة الايونات و تركيز كل من الالكترونات والبوزترونات) على الموجات الصدمية الصوتية المغناطيسية. نتائج الدراسة تشير إلى أن خواص الموجات الصدمية تتأثر بشكل ملحوظ بتلك البارامترات. نتائج هذه الدراسة قد تساهم في فهم خواص الموجات الصوتية المغناطيسية المنتشرة في بيئات بلازما الفضاء الكثيفة والتي يكون فيها حضور للالكترونات والبوزترونات الكمية.

1.Introduction

Over the past decade, quantum plasmas have gained considerable attention due to its potential applications in the microelectronic devices [1], nanoscale systems [2], laser plasmas [3] and in dense astrophysical plasmas [4-7]. In fact,the study of quantum plasma becomes important when the thermal de Broglie wavelength associated with the plasma particles becomes of the order or greater than the particle Debye length, and the plasma behaves like a Fermi gas. In such situations, the quantum mechanical effects play a significant role in the collective behavior of plasma particles [8–11]. It is well-known that, when a plasma is immersed in an external magnetic field, there exists the possibility of a new class of wave modes with a frequency much less than the plasma frequency, such as Alfven waves [12,13] and magnetoacoustic (MA) waves (or magnetosonic waves) [14]. The magnetosonic wave is a fundamental mode of electromagnetic wave propagating perpendicular to the ambient magnetic field in a plasma media. It arises due to ion inertia, which provides the

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a E-mail addresses: i.luqman@su.edu.ye
b Corresponding author .E-mail addresses: mahkhaled@hotmail.com
c E-mail addresses: ka.alkuhlani@gmail.com

inertial force and the restoring force comes from the compression of both magnetic field and density of plasmas.

Generally, applicable theory to study the behavior of MA waves in a quantum magneto-plasma is usually referred to quantum magneto hydrodynamic (QMHD) theory, which is a natural extension to the classical theory of magneto hydrodynamics (MHD) that is used to describe the conventional magneto-plasma fluid. In the recent years, many authors have examined the linear and nonlinear characteristics of the MA waves in the framework of the QMHD approximation in different quantum plasma systems. For example, Marklund et al. [15] studied the magnetosonic solitons in a Fermionic quantum magneto-plasma including the effects of quantum Bohm potential and electron spin-1/2. They found that the system of the QMHD equations admit rarefactive solitons due to the balance between nonlinearities and quantum diffraction effects. Shukla [16] investigated the linear MA waves in a quantum magneto-dusty plasma, considering the effects of quantum Bohm potential and electron spin-1/2. Masood [17] derived quantum Kadomtsev–Petviashvili–Burgers equation and studied the MA shock waves in two dimensional quantum plasma. Mushtaq and Qamar [18] studied nonlinear magnetosonic waves in quantum plasmas with/without spin effects using the QMHD model. Hussain and Mahmood [19] studied the MA shock waves in electron-ion quantum plasma by using the QMHD model taking into account the kinematic viscosity of the ions and quantum Bohm potential. Hussain et al. [20] investigated the nonlinear propagation of the MA waves in quantum electron-positron-ion plasmas. They found that the concentration of positrons has significant impact on the dispersive properties of the fast MA wave.

However, in order to fully understand the properties of MA waves in quantum magneto-plasma, the exchange-correlation potential effects should be taken into account, especially when spin magnetization effects are present in the system [21-25]. In fact, the source of the exchange-correlation potential is as follows: the interaction of the quantum plasma particles can be separated into a Hartree term (due to the electrostatic potential of the number density of plasma particles) and a particle exchange potential due to the electron spin-$1/2$ effect. Sahu and Misra [26] investigated nonlinear propagating of the magnetosonic shock waves in a dissipative quantum magneto-plasma consisting of a quantum electron and classical viscous ions, including the electron exchange-correlation effects. They used the QMHD model and found the exchange-correlation effects are more dominant and responsible in the transition from monotonic to oscillatory shocks to other quantum effects.

The aim of this article is to investigate the characteristics of the nonlinear excitations of the MA waves in quantum electrons-positrons-ions magneto-plasma, considering the contributions of the spin-1/2 and the exchange-correlation effects for both electrons and positrons. For ions, we neglect their quantum contributions because of large inertia. The outline of this paper is as follows: the basic equations governing the quantum magneto-plasma system under consideration are presented in Sect. 2. Derivation of Korteweg-devris-Burger (KdVB) equation is given in Sect. 3. Analytical solutions of the KdV-B equation is given in Sect. 4. While numerical results and discussion are provided in Sect. 5. Sect. 6 is devoted to conclusions.

2. Governing Equations

We consider a collision-less electron-positron-ion plasma placed in an external magnetic field, along the z direction as $\boldsymbol{B_0} = B_0\boldsymbol{e_z}$ where $\boldsymbol{e_z}$ is the unit vector along the z-axis. The electrons and positrons are assumed to be inertialless and degenerate having spin and exchange-correlation effects. While the ions are taken to be inertial and classical. Incidentally, in dense astrophysical environments, the Fermi pressure for the ions is negligible as compared to that for the electrons and positrons. So the pressure effects are neglected for the ions, whereas the electrons and positrons are assumed to obey the equation of state for a zero temperature Fermi gas. Thus, the continuity and momentum equations governing the dynamics of the ions are respectively given by

$$\frac{\partial n_i}{\partial t} + \boldsymbol{\nabla}\cdot(n_i\boldsymbol{u}_i) = 0, \tag{1}$$

$$m_i n_i\left(\frac{\partial}{\partial t} + \boldsymbol{u}_i\cdot\boldsymbol{\nabla}\right)\boldsymbol{u}_i = en_i(\mathbf{E} + \boldsymbol{u}_i\times\boldsymbol{B}) + \eta_d\nabla^{\boldsymbol{2}}\boldsymbol{u}_i \tag{2}$$

where $n_i(\boldsymbol{u}_i)$ is the density (velocity) number of the ions, m_i is the ion mass, $\mathbf{E}$ is the self-consistent electric field and η_d is the dynamic viscosity of the ions. The continuity and momentum equations for the electron/positrons are given by

$$\frac{\partial n_j}{\partial t} + \boldsymbol{\nabla} \cdot (n_j \boldsymbol{u}_j) = 0, \tag{3}$$

$$0 = q_j n_j (\mathbf{E} + \boldsymbol{u}_j \times \boldsymbol{B}) - \boldsymbol{\nabla} P_{Fj} + \boldsymbol{F}_{Qj}, \tag{4}$$

where $n_j(\boldsymbol{u}_j)$ is the density and velocity number of plasma particles (electrons or positrons) and $P_{Fj} = (2\mathcal{E}_{Fj} n_{j0}^{-2/3}/5) n_j^{5/3}$ is the Fermi pressure due to the Fermionic nature of the plasma particles with n_{j0} is the equilibrium density of the plasma particles, $\mathcal{E}_{Fj} = k_B T_{Fj}$ is the Fermi energy, $T_{Fj} = \hbar^2 (3\pi^2 n_{j0})^{2/3}/2m\, k_B$ is the Fermi temperature, $m(= m_e = m_p)$ is the electron/positron mass, k_B is the Boltzmann constant, $\hbar$ is Planck's constant divided by 2π. Here, the label j characterizes the electrons ($j = e$, with $q_e = -e$), and the positrons ($j = p$, with $q_p = e$). The last term in Eq. (4) ($\boldsymbol{F}_{Qj}$) represents the total quantum force on the plasma particles i.e. electrons ($j = e$) or positrons ($j = p$), which can be expressed as

$$\boldsymbol{F}_{Qj} = \frac{\hbar^2 n_j}{2m} \boldsymbol{\nabla} \left(\frac{1}{\sqrt{n_j}} \nabla^2 \sqrt{n_j} \right) + n_j \mu_B L_j(\varepsilon_j) \nabla B - n_j \boldsymbol{\nabla} V_j^{xc}. \tag{5}$$

The first term in Eq. (5) is the gradient of the so called Bohm potential, the second term comes from the spin-$1/2$ of electron or positron where $\mu_B = e\hbar/2m$ is the "Bohr magneton" and $B = |\boldsymbol{B}|$. Here, $L_j(\varepsilon_j) = \tanh(\varepsilon_j)$ is the Langevin function, which is due to the magnetization of a spin distribution of the plasma particles in thermodynamic equilibrium where $\varepsilon_j = \mu_B B/\mathcal{E}_{Fj}$ is the ratio of the Zeeman energy of the magnetic moment in the external field to the Fermi energy ($\mathcal{E}_{Fj}$) (also called the magnetization energy). In the most dense plasma situations, the condition $\mu_j B \ll \mathcal{E}_{Fj}$ (or $\varepsilon_j \ll 1$) is satisfied, and thus we can use the approximation $\tanh(\varepsilon_j) \approx \varepsilon_j = \mu_j B/k_B T_{Fj}$ [27]. The last term in Eq. (5) is the exchange-correlation potential gradient force where V_j^{xc} denotes the exchange-correlation potential of the electrons ($j = e$) or positrons ($j = p$),which is given by Eq. (8) and can be simplify to $V_j^{xc} \approx -1.62(e^2/4\pi\epsilon_0) n_j^{1/3} + 5.65(\hbar^2/m) n_j^{2/3}$ [26].

The Maxwell's equations are given by

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{6}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 (\mathbf{J} + \mathbf{J}_M) + \frac{1}{c^2} \frac{\partial \boldsymbol{E}}{\partial t}, \tag{7}$$

where $\boldsymbol{E}$ is the electric filed vector, $c = 1/\sqrt{\mu_0 \epsilon_0}$ is the light speed in a vacuum, μ_0 is the permeability of free space, $\mathbf{J} = e(n_i \boldsymbol{u}_i + n_p \boldsymbol{u}_p - n_e \boldsymbol{u}_e)$ is the true electric current densities and $\mathbf{J}_M = \boldsymbol{\nabla} \times \boldsymbol{M}$ is the magnetization spin current densities of the plasma particles (electrons and positrons). Here,$\boldsymbol{M} = \boldsymbol{M_e} + \boldsymbol{M_p}$ where $\boldsymbol{M_e}$ and $\boldsymbol{M_p}$ are the magnetization of electrons and positrons respectively. $\boldsymbol{M} = 2 q_j n_j \mu_B \boldsymbol{S}/\hbar |q_j|$ for each species, where $\boldsymbol{S}$ is the spin vector. In the limiting case where the time-scales are much longer than the Larmor period, the spin vector can be approximated by $\boldsymbol{S} = \hbar q_j \tanh(\mu_B B/\mathcal{E}_{Fj})\, \widehat{\boldsymbol{b}}/2|q_j|$, $\widehat{\boldsymbol{b}} = \boldsymbol{B}/|\boldsymbol{B}|$ is a unit vector in the direction of the magnetic field.

Now, to simplify the above equations further, we use the following non-dimensional variables:

$$n_j \to \frac{n_j}{n_{j0}}, \qquad r \to \left(\frac{\omega_{ci}}{V_A}\right) r, \;\; t \to \omega_{ci} t, \qquad \boldsymbol{u}_{i,j} \to \frac{\boldsymbol{u}_{i,j}}{V_A}, \qquad \boldsymbol{B} \to \frac{\boldsymbol{B}}{\boldsymbol{B}_0},$$

$$\boldsymbol{E} \to \frac{\boldsymbol{E}}{V_A B_0}, \qquad \boldsymbol{M} \to \frac{\mu_0}{B_0} \boldsymbol{M},$$

where $\omega_{ci} = eB_0/m_i$ is the ion cyclotron frequency and $V_A = B_0/\sqrt{\mu_0 m_i n_{i0}}$ is the Alfven speed. Using the above relations into the Eqs. (1)-(7), the normalized basic equations can be written in the following form:

$$\frac{\partial n_i}{\partial t} + \boldsymbol{\nabla} \cdot (n_i \boldsymbol{u}_i) = 0, \tag{8}$$

$$n_i \left(\frac{\partial \boldsymbol{u}_i}{\partial t} + (\boldsymbol{u}_i \cdot \boldsymbol{\nabla}) \boldsymbol{u}_i \right) = n_i (\mathbf{E} + \boldsymbol{u}_i \times \boldsymbol{B}) + \eta \nabla^{\mathbf{2}} \boldsymbol{u}_i \tag{9}$$

$$\frac{\partial n_j}{\partial t} + \boldsymbol{\nabla} \cdot (n_j \boldsymbol{u}_j) = 0, \tag{10}$$

$$0 = -\mathbf{E} - \boldsymbol{u}_e \times \boldsymbol{B} - \frac{\beta}{2} \boldsymbol{\nabla} n_e^{2/3} + \frac{H^2}{2} \boldsymbol{\nabla} \left(\frac{\nabla^2 \sqrt{n_e}}{\sqrt{n_e}} \right) + \beta \varepsilon_0^2 B \boldsymbol{\nabla} B + \beta \alpha \boldsymbol{\nabla} n_e^{1/3} - \beta \gamma \boldsymbol{\nabla} n_e^{2/3}, \tag{11}$$

$$0 = \mathbf{E} + \boldsymbol{u}_p \times \boldsymbol{B} - \frac{\sigma \beta}{2} \boldsymbol{\nabla} n_p^{2/3} + \frac{H^2}{2} \boldsymbol{\nabla} \left(\frac{\nabla^2 \sqrt{n_p}}{\sqrt{n_p}} \right) + \frac{\beta \varepsilon_0^2}{\sigma} B \boldsymbol{\nabla} B + p^{1/3} \beta \alpha \boldsymbol{\nabla} n_p^{1/3} - p^{2/3} \beta \gamma \boldsymbol{\nabla} n_p^{2/3}, \tag{12}$$

where $\varepsilon_0 = \mu_B B_0 / \mathcal{E}_{Fe}$ is the normalized Fermi-Zeeman energy, $p = n_{p0}/n_{e0}$ $(0 < p < 1)$ is the equilibrium density ratio of positron-to-electron, $\sigma = T_{Fp}/T_{Fe}$ is the Fermi temperature ratio of positron-to-electron which is related to p by $\sigma = p^{2/3}$. $\beta = C_{is}^2/V_A^2 = 2\mu_0 n_{i0} \mathcal{E}_{Fe}/B_0^2$ is the plasma beta with $C_{is} = (2\mathcal{E}_{Fe}/m_i)^{1/2}$is the quantum ion acoustic speed. $H = \omega_{ci}\hbar/\sqrt{mm_i}V_A^2$ represents the normalized quantum parameter, $\delta = V_A^2/c^2$ and $\eta = \eta_d \omega_{ci}/m_i n_{i0} V_A^2$ is the normalized viscosity coefficient. Here, γ and α represent the exchange-correlation coefficients, which are given by $\gamma = 5.65\left(\hbar^2 n_{e0}^{2/3}/2mE_{Fe}\right) \approx 0.59$ and $\alpha = 1.62\left(e^2 n_{e0}^{1/3}/8\pi\epsilon_0 E_{Fe}\right)$, and the normalized magnetization density $\boldsymbol{M}$ is defined by $\boldsymbol{M} = [\beta \varepsilon_0^2 / 2\sigma(1-p)]\left(\sigma n_e + p n_p\right)\boldsymbol{B}$.

3. Derivation of Korteweg-devris Burger Equation

In order to derive the Korteweg-devris Burger (KdVB) equation, we use the standard reductive perturbation method [28] consider the propagation wave in x-direction and thus $\nabla = (\partial/\partial x, 0, 0)$, $\boldsymbol{u}_s = \left(u_{sx}, u_{sy}, u_{sz}\right)$ where $s = i$ for ions, $s = e$ for electrons and $s = p$ for positrons. Therefore the stretched variables are defined as:

$$\xi = \epsilon^{1/2}\left(x - V_p t\right), \qquad \tau = \epsilon^{3/2} t, \tag{13}$$

where ϵ is a small expansion parameter which lies in the range $0 < \epsilon < 1$ and V_p is the normalized phase velocity of the wave to be determined later. The perturbed quantities are expanded in terms of the smallness parameter ϵ in the following form:

$$\begin{pmatrix} n_s \\ u_{sx} \\ B \\ E_y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \epsilon \begin{pmatrix} n_s^{(1)} \\ u_{sx}^{(1)} \\ B^{(1)} \\ E_y^{(1)} \end{pmatrix} + \epsilon^2 \begin{pmatrix} n_s^{(2)} \\ u_{sx}^{(2)} \\ B^{(2)} \\ E_y^{(2)} \end{pmatrix} + \cdots, \tag{14}$$

$$\begin{pmatrix} u_{sy} \\ E_x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \epsilon^{3/2} \begin{pmatrix} u_{sy}^{(1)} \\ E_x^{(1)} \end{pmatrix} + \epsilon^{5/2} \begin{pmatrix} u_{sy}^{(2)} \\ E_x^{(2)} \end{pmatrix} + \cdots. \tag{15}$$

Substituting Eqs. (13) –(15) into Eqs. (8)-(12) and collecting the lowest order terms ($\epsilon^{3/2}$) of the ion continuity and x, y components of the ion momentum equation give the following set of equations:

$$-V_p \frac{\partial n_i^{(1)}}{\partial \xi} + \frac{\partial u_{ix}^{(1)}}{\partial \xi} = 0, \tag{16}$$

$$E_x^{(1)} + u_{iy}^{(1)} + V_p \frac{\partial u_{ix}^{(1)}}{\partial \xi} = 0, \tag{17}$$

$$E_y^{(1)} - u_{ix}^{(1)} = 0. \tag{18}$$

The lowest order terms ($\epsilon^{3/2}$) of continuity equation and x, y components of momentum equation of electrons give the following set of equations:

$$-V_p \frac{\partial n_e^{(1)}}{\partial \xi} + \frac{\partial u_{ex}^{(1)}}{\partial \xi} = 0, \tag{19}$$

$$E_x^{(1)} + u_{ey}^{(1)} + \frac{\beta(1-\alpha_e)}{3}\frac{\partial n_e^{(1)}}{\partial \xi} - \beta\varepsilon_0^2 \frac{\partial B^{(1)}}{\partial \xi} = 0, \tag{20}$$

$$E_y^{(1)} - u_{ex}^{(1)} = 0, \tag{21}$$

and the lowest order terms ($\epsilon^{3/2}$) of positron continuity equation and x, y component of momentum equation of positrons are

$$-V_p \frac{\partial n_p^{(1)}}{\partial \xi} + \frac{\partial u_{px}^{(1)}}{\partial \xi} = 0, \tag{22}$$

$$E_x^{(1)} + u_{py}^{(1)} - \frac{\beta(\sigma-\alpha_p)}{3}\frac{\partial n_p^{(1)}}{\partial \xi} + \frac{\beta\varepsilon_0^2}{\sigma} \frac{\partial B^{(1)}}{\partial \xi} = 0, \tag{23}$$

$$E_y^{(1)} - u_{px}^{(1)} = 0, \tag{24}$$

where $\alpha_e = (\alpha - 2\gamma)$ and $\alpha_p = p^{1/3}\left(\alpha - 2p^{1/3}\gamma\right)$. The lowest order terms ($\epsilon^{3/2}$) of Faraday's law gives

$$\frac{\partial E_y^{(1)}}{\partial \xi} - V_p \frac{\partial B^{(1)}}{\partial \xi} = 0. \tag{25}$$

The lowest order (ϵ) terms of x and y component of Ampere's law is described as

$$u_{ix}^{(1)} + \frac{p}{1-p}u_{px}^{(1)} - \frac{1}{1-p}u_{ex}^{(1)} = 0, \tag{26}$$

$$\left(1+\frac{\Sigma_0}{1-p}\right)\frac{\partial B^{(1)}}{\partial \xi} + \frac{\beta\varepsilon_0^2}{2(1-p)}\frac{\partial n_e^{(1)}}{\partial \xi} + \frac{\beta\varepsilon_0^2 p}{2\sigma(1-p)}\frac{\partial n_p^{(1)}}{\partial \xi} - \frac{1}{1-p}u_{ey}^{(1)} + \frac{p}{1-p}u_{py}^{(1)} + u_{iy}^{(1)} - \delta V_p \frac{\partial E_y^{(1)}}{\partial \xi} = 0, \tag{27}$$

where $\Sigma_0 = \beta\varepsilon_0^2(\sigma+p)/2\sigma$. From equations (18), (21), (24) and (25), we obtain

$$E_y^{(1)} = u_{ex}^{(1)} = u_{px}^{(1)} = u_{ix}^{(1)} = V_p B^{(1)}, \tag{28}$$

Using the above equation with the equations (16), (19) and (22), we get

$$n_i^{(1)} = n_e^{(1)} = n_p^{(1)} = B^{(1)} = \frac{u_{ix}^{(1)}}{V_p}. \tag{29}$$

Substituting Eqs. (28) and (29) into Eqs. (17), (20), (23) and (27), we get

$$u_{iy}^{(1)} = -V_p \frac{\partial u_{ix}^{(1)}}{\partial \xi} - E_x^{(1)}, \tag{30}$$

$$u_{ey}^{(1)} = -\frac{\beta}{3V_p}(1-\alpha_e-3\varepsilon_0^2)\frac{\partial u_{ix}^{(1)}}{\partial \xi} - E_x^{(1)}, \tag{31}$$

$$u_{py}^{(1)} = \frac{\beta}{3V_p}\left(\sigma-\alpha_p-\frac{3\varepsilon_0^2}{\sigma}\right)\frac{\partial u_{ix}^{(1)}}{\partial \xi} - E_x^{(1)}, \tag{32}$$

$$\frac{1}{V_p}\left(1-\delta V_p^2 - \frac{2\Sigma_0}{1-p}\right)\frac{\partial u_{ix}^{(1)}}{\partial \xi} + u_{iy}^{(1)} + \frac{p}{1-p}u_{py}^{(1)} - \frac{1}{1-p}u_{ey}^{(1)} = 0. \tag{33}$$

Substituting Eqs. (30)–(32) into the Eq. (33), the linear phase velocity of the MA wave is obtained as follows:

$$V_p = \sqrt{\frac{(1-p)+\beta(p\sigma+1-\vartheta)/3-4\Sigma_0}{(1+\delta)(1-p)}}, \tag{34}$$

where $\vartheta = p\alpha_p + \alpha_e$. It can be noted from Eq. (34) that the linear phase velocity V_p is modified due to the presence of spin-$1/2$ and exchange-correlation potential effects.

To the next higher order ($\epsilon^{5/2}$) terms of ion continuity and x, y components of ion momentum equations, we get the following set of equations:

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V_p \frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial u_{ix}^{(2)}}{\partial \xi} + \frac{\partial n_i^{(1)} u_{ix}^{(1)}}{\partial \xi} = 0, \tag{35}$$

$$\frac{\partial u_{ix}^{(1)}}{\partial \tau} - V_p n_i^{(1)} \frac{\partial u_{ix}^{(1)}}{\partial \xi} - V_p \frac{\partial u_{ix}^{(2)}}{\partial \xi} + u_{ix}^{(1)} \frac{\partial u_{ix}^{(1)}}{\partial \xi} - E_x^{(2)} - u_{iy}^{(2)} - n_i^{(1)}\left(E_x^{(1)} + u_{iy}^{(1)}\right) - B^{(1)} u_{iy}^{(1)} - \eta_0 \frac{\partial^2 u_{ix}^{(1)}}{\partial \xi^2} = 0, \tag{36}$$

$$V_p \frac{\partial u_{iy}^{(1)}}{\partial \xi} + \left(E_y^{(2)} - u_{ix}^{(2)}\right) - B^{(1)} u_{ix}^{(1)} = 0. \tag{37}$$

The next higher order terms ($\epsilon^{3/2}$) of electron continuity and x, y components of electron momentum equations give the following equations:

$$\frac{\partial n_e^{(1)}}{\partial \tau} - V_p \frac{\partial n_e^{(2)}}{\partial \xi} + \frac{\partial u_{ex}^{(2)}}{\partial \xi} + \frac{\partial n_e^{(1)} u_{ex}^{(1)}}{\partial \xi} = 0, \tag{38}$$

$$E_x^{(2)} + u_{ey}^{(2)} + B^{(1)} u_{ey}^{(1)} - \frac{\beta}{9}(1 + 2\gamma - 2\alpha) n_e^{(1)} \frac{\partial n_e^{(1)}}{\partial \xi} + \frac{\beta}{3}(1 - \alpha_e) \frac{\partial n_e^{(2)}}{\partial \xi} - \frac{H^2}{4} \frac{\partial^3 n_e^{(1)}}{\partial \xi^3} - \beta \varepsilon_0^2 \frac{\partial B^{(2)}}{\partial \xi} - \beta \varepsilon_0^2 B^{(1)} \frac{\partial B^{(1)}}{\partial \xi} = 0. \tag{39}$$

$$E_y^{(2)} = u_{ex}^{(2)} + u_{ex}^{(1)} B^{(1)}. \tag{40}$$

The next higher order ($\epsilon^{3/2}$) terms of positron continuity and x, y components of positron momentum equations give the following set of equations:

$$\frac{\partial n_p^{(1)}}{\partial \tau} - V_p \frac{\partial n_p^{(2)}}{\partial \xi} + \frac{\partial u_{px}^{(2)}}{\partial \xi} + \frac{\partial n_p^{(1)} u_{px}^{(1)}}{\partial \xi} = 0, \tag{41}$$

$$E_x^{(2)} + u_{py}^{(2)} + B^{(1)} u_{py}^{(1)} + \frac{\beta}{9}\left(\sigma + 2p^{2/3}\gamma - 2p^{1/3}\alpha\right) n_p^{(1)} \frac{\partial n_p^{(1)}}{\partial \xi} + \frac{H^2}{4} \frac{\partial^3 n_p^{(1)}}{\partial \xi^3} - \frac{\beta}{3}\left(\sigma - \alpha_p\right) \frac{\partial n_p^{(2)}}{\partial \xi} + \frac{\beta \varepsilon_0^2}{\sigma} B^{(1)} \frac{\partial B^{(1)}}{\partial \xi} + \frac{\beta \varepsilon_0^2}{\sigma} \frac{\partial B^{(2)}}{\partial \xi} = 0, \tag{42}$$

$$E_y^{(2)} = u_{px}^{(2)} + u_{px}^{(1)} B^{(1)}. \tag{43}$$

The next higher order (ϵ^2) terms of x component of Ampere's law gives

$$\delta V_p \frac{\partial E_x^{(1)}}{\partial \xi} + \frac{1}{1-p} u_{ex}^{(2)} + \frac{1}{1-p} n_e^{(1)} u_{ex}^{(1)} - u_{ix}^{(2)} - n_i^{(1)} u_{ix}^{(1)} - \frac{p}{1-p} u_{px}^{(2)} - \frac{p}{1-p} n_p^{(1)} u_{px}^{(1)} = 0. \tag{44}$$

The higher order ($\epsilon^{5/2}$) terms of Faraday's law and y component of Ampere's law yields

$$\frac{\partial B^{(1)}}{\partial \tau} - V_p \frac{\partial B^{(2)}}{\partial \xi} + \frac{\partial E_y^{(2)}}{\partial \xi} = 0, \tag{45}$$

$$u_{iy}^{(2)} + n_i^{(1)} u_{iy}^{(1)} + \frac{p}{1-p} u_{py}^{(2)} + \frac{p}{1-p} n_p^{(1)} u_{py}^{(1)} - \frac{1}{1-p} u_{ey}^{(2)} - \frac{1}{1-p} n_e^{(1)} u_{ey}^{(1)} + \left(1 - \frac{\Sigma_0}{1-p}\right) \frac{\partial B^{(2)}}{\partial \xi} - \frac{\beta \varepsilon_0^2}{2(1-p)} \frac{\partial n_e^{(2)}}{\partial \xi} - \frac{\beta \varepsilon_0^2 p}{2\sigma(1-p)} \frac{\partial n_p^{(2)}}{\partial \xi} - \frac{2\Sigma_0}{(1-p)} u_{ix}^{(1)} \frac{\partial u_{ix}^{(1)}}{\partial \xi} - \delta V_p \frac{\partial E_y^{(2)}}{\partial \xi} + \delta \frac{\partial E_y^{(1)}}{\partial \tau} = 0. \tag{46}$$

Finally, eliminating, $u_{iy}^{(2)}$, $u_{ey}^{(2)}$, $u_{py}^{(2)}$ and $E_x^{(2)}$, $B^{(2)}$, $n_j^{(2)}$, $u_{jx}^{(2)}$ from Eqs. (35)–(46) and using relations given in Eqs. (28)-(33), we obtain KdVB equation for MA waves in a dissipative quantum plasma described as follows:

$$\frac{\partial u_{ix}^{(1)}}{\partial \tau} + Q u_{ix}^{(1)} \frac{\partial u_{ix}^{(1)}}{\partial \xi} + G \frac{\partial^3 u_{ix}^{(1)}}{\partial \xi^3} - R \frac{\partial^2 u_{ix}^{(1)}}{\partial \xi^2} = 0, \tag{45}$$

where the coefficient of nonlinearity (i.e., Q) is given by

$$Q = \frac{1}{2V_p^2(1-p)(1+\delta)} \left\{ \frac{\beta}{9} \left[5(p\sigma + 1 - \vartheta) + \alpha\left(p^{4/3} + 1\right) \right] - 12\Sigma_0 + 2(1-p) + 3(1-p)V_p^2 - 2(1-p)(1+\delta)V_p^2 \right\}. \tag{46}$$

The dispersion coefficient (i.e., G) is given as

$$G = \frac{1}{2(\delta+1)} \left[\frac{\delta V_p^3}{(1+\delta)} - \frac{p+1}{(1-p)} \frac{\mathcal{H}^2}{4V_p} \right], \tag{47}$$

and the dissipation coefficient (i.e., R) is given as

$$R = \frac{\eta_0}{2(\delta+1)}. \tag{48}$$

4. Analytical Solutions of KdVB Equation

The nonlinear KdVB equation (45) describes the weakly nonlinear MA wave when the plasma system has both dispersive and dissipative effects. In order to obtain an analytical solution of this equation, we introduce the transformations

$$\chi = \xi - U_0 \tau, \qquad u_{ix}^{(1)}(\xi, \tau) = V(\chi) \tag{49}$$

where χ is the transformed coordinates with respect to a frame moving with velocity U_0. Using the transformations (49) in the KdVB equation (45) we get

$$-U_0 \frac{dV}{d\chi} + QV \frac{dV}{d\chi} + G \frac{d^3 V}{d\chi^3} - R \frac{d^2 V}{d\chi^2} = 0. \tag{50}$$

Integrating Eq. (50), with using the boundary conditions of $V, dV/d\chi$, and $d^2V/d\chi^2 \to 0$ as $\chi \to \pm\infty$, we get

$$G \frac{d^2 V}{d\chi^2} - R \frac{dV}{d\chi} + \frac{Q}{2} V^2 - U_0 V = 0. \tag{52}$$

Now, we first consider the non-dissipative case (i.e. $R = 0$), which leads to the following energy equation

$$\frac{1}{2} \left(\frac{dV}{d\chi} \right)^2 + \frac{Q}{6P} V^3 - \frac{U_0}{2P} V^2 = 0, \tag{53}$$

Using the boundary conditions $V = dV/d\chi = 0$ at $\chi \to \pm\infty$, the solitary wave solution of Eq. (53) is

$$V = V_m \operatorname{sech}^2 \left(\frac{\chi}{\Delta} \right), \tag{54}$$

where $V_m = 3U_0/Q$ and $\Delta = 2\sqrt{G/U_0}$ represent the amplitude and width of the wave, respectively.

From Eqs. (46) and (47), we find that the coefficients Q and G are modified by the inclusion of exchange-correlation and spin-1/2 effects. This means that the profile of the MA solitary wave (the amplitude, width and phase velocity) is modified by the inclusion of exchange-correlation (via parameters α and γ) and spin-1/2 (via the normalized Zeeman energy ε_0) as well as the other plasma parameters such as p, σ and plasma beta β. Only the width of the MA solitary wave is modified by the inclusion of quantum Bohm potential (via the quantum diffraction H). Now, after indicating the solitary wave solution which arises from neglecting the Burger term, we take into account the dissipation effect, i.e. $R \neq 0$, and its resultant shock waves. The tanh-method [29] is employed to investigate the shock wave structure of Eq. (45). Now, we introduce a new independent variable $Y = \tanh(\chi)$, due to which Eq. (52) transforms to

$$G(1-Y^2)^2\frac{d^2V}{dY^2}-(1-Y^2)(R+2GY)\frac{dV}{dY}+\frac{Q}{2}V^2-U_0V=0. \tag{55}$$

It is obvious that, Eq. (55) is a Fuchsian-like nonlinear ordinary differential equation. Assume the series solution in the form

$$V(Y)=\sum_{j=0}^{N}a_jY^j. \tag{56}$$

The upper limit (N) can be determined by the subtle balance method. According to this method, balancing the highest order nonlinear term that has the exponent $2N$, with the highest order derivative that has the exponent $N+2$, in Eq. (52) yields $2N=N+2$ that gives $N=2$. Therefore, the finite power series solution in terms of Y can be expressed as

$$V(Y)=a_0+a_1Y+a_2Y^2. \tag{57}$$

Substituting Eq.(57) into Eq.(55), collecting the coefficients of each power of Y^r, $0\le r\le 8$, setting each coefficient to zero, the unknowns parameters a_0, a_1 and a_2 are determined as

$$a_0=\frac{1}{Q}(U_0+12G),\qquad a_1=-\frac{12R}{5Q},\qquad a_2=-\frac{12G}{Q}. \tag{58}$$

Therefore, the analytical solution of the KdVB equation is given by

$$V(\chi)=\frac{1}{Q}\left[U_0-\frac{12R}{5}\tanh(\chi)+12G\,\mathrm{sech}^2(\chi)\right]. \tag{59}$$

The correlation between dissipation and dispersion terms participates strongly in structuring the shock wave. When the dissipation effect is the most prominent, the dispersive term should be disregarded in Eq. (52) (i.e., $G\to 0$), and accordingly, the KdVB equation (45) redacts to Burger's equation, and then Eq. (52) can be rewritten as follows

$$R\frac{dV}{d\chi}=\frac{Q}{2}V^2-U_0V. \tag{60}$$

Equation (60) admits a monotonic shock wave solution of the form

$$V=\frac{U_0}{Q}\left[1-\tanh\left(\frac{U_0}{2R}\chi\right)\right], \tag{61}$$

where $\chi=\xi-U_0\tau$. It is clear that, the above analytical solution exhibits only astrictly monotonic shock structure**.** On the other hand**,** to discuss the oscillatory shock wave profiles, we assume that the solution of Eq. (52) has the form $V(\chi)=V_c+V'(\chi)$, where V_c represents the shock amplitude with $|V'|\ll|V_c|$. Using this solution in Eq. (52) and then linearize it with respect to V', the following equation is obtained

$$\frac{d^2V'}{d\chi^2}-\frac{R}{G}\frac{dV'}{d\chi}+\frac{U_0}{G}V'=0. \tag{62}$$

To get the shock amplitude V_c, we set the boundary conditions $V=V_c$ and $dV/d\chi=d^2V/d\chi^2=0$ as $\chi\to-\infty$ in Eq. (52). Then, we obtain

$$V_c=\frac{2U_0}{Q}. \tag{63}$$

Equation (62) is a standard type of differential equation, and the procedure for solving it goes as follows: Look for solutions of the form $V'(\chi)=\exp(\lambda\chi)$ where λ is a constant, by substituting this solution into Eq. (62), we obtain that λ must satisfy the characteristic equation

$$\lambda^2-\frac{R}{G}\lambda+\frac{U_0}{G}=0, \tag{64}$$

which has the solutions

$$\lambda_{1,2}=\frac{R}{2G}\pm\frac{1}{2G}\sqrt{R^2-4U_0G}, \tag{65}$$

where λ_1 and λ_2 may be either real, or complex conjugates depending on the sign of (R^2-4GU_0). Here, we consider only the limiting case when $(R^2-4GU_0)<0$, $\lambda_{1,2}=(\mathrm{R}/2\mathrm{G})\pm i\Lambda$ are complex conjugates, where $\Lambda=\sqrt{(4GU_0-R^2)}/2G$ is a real number and $i=\sqrt{-1}$. The general complex solution then becomes:

$$V' = \exp\left(\frac{R}{2G}\chi\right)[C_1 \exp(i\Lambda\chi) + C_2 \exp(-i\Lambda\chi)], \tag{66}$$

where C_1 and C_2 are complex constants, which are complex. We put $C_1 = Ce^{i\mu}/2$ and $C_2 = Ce^{-i\mu}/2$, where $C = 2|C_1|$. Then, the solution (66) reduces to the alternative form

$$V' = C\exp\left(\frac{R}{2G}\chi\right)\cos(\Lambda\chi + \mu). \tag{67}$$

Thus, the oscillatory shock wave solution is

$$V = \frac{2U_0}{Q} + C\exp\left(\frac{R}{2G}\chi\right)\cos(\Lambda\chi + \mu). \tag{68}$$

where C and μ are real arbitrary constants with $C > 0$ and μ is the phase angle.

5. Results and Discussion

In this section, we have examined the nonlinear propagation characteristics of the MA solitary and shock waves in a quantum mgnato-plasma model containing ions, and degenerate electrons and positrons taking into account the spin- $1/2$ and exchange-correlation effects. In the dense astrophysical objects such as neutron stars and white dwarfs, used in the following numerical illustration, are chosen as follows [30-32]: $n_{e0} = (1-2)\times 10^{34}m^{-3}$, $B_0 = (0.1-1)\times 10^7 T$, $p = 0.3 - 0.7$ and $n_{i0} = (1-p)n_{e0}$. As we mentioned in Sect. 4, if the dissipation is negligible, the MA solitary waves will appear in the medium due to the balance between dispersive and nonlinear terms. On the other hand, the MA shock waves (both monotonic and oscillatory types) appear in the system as a result of dissipative due to the dynamic viscosity. Figure 1 shows the monotonic shock solution given by Eq.(61) for different values of positrons concentration (via the parameter p). It is noticed from this figure that the increase in the value of p leads to increasing the magnitude of the amplitude of monotonic shock waves. Physically, this means that the increase in the positrons concentration leads to a decrease in the nonlinearity coefficient, which in turn leads to an increase in the amplitude of monotonic shock U_0/Q. The effect of external the magnetic field(via B_0) on the monotonic shock profile of the MA waves is shown in Fig. 2. It is noticed that by increasing the value of the magnetic field strength, the amplitude of monotonic shock waves increases.

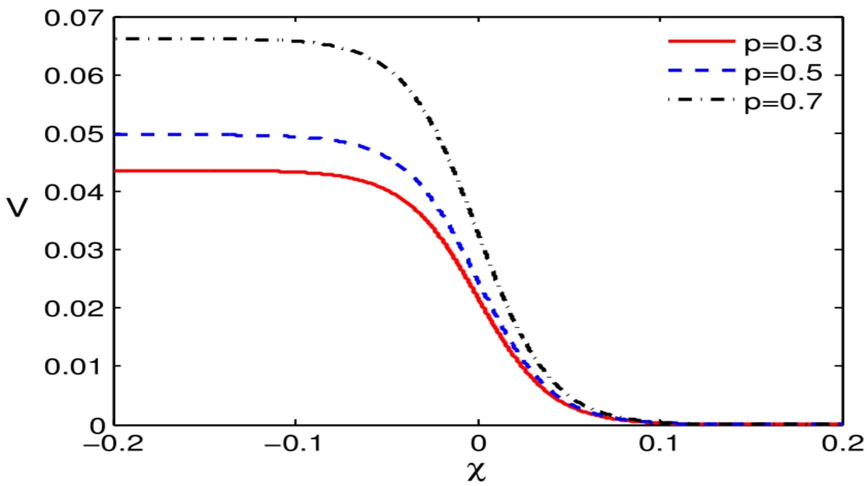

Figure 1: The magnetoacoustic shock waves against χ for different values of p with $n_{e0} = 10^{34}$, $B_0 = 0.6\times 10^7\ T$, $\eta_0 = 0.02$, $\gamma = 0.59$, $\alpha = 0.015$ and $U_0 = 0.5$.

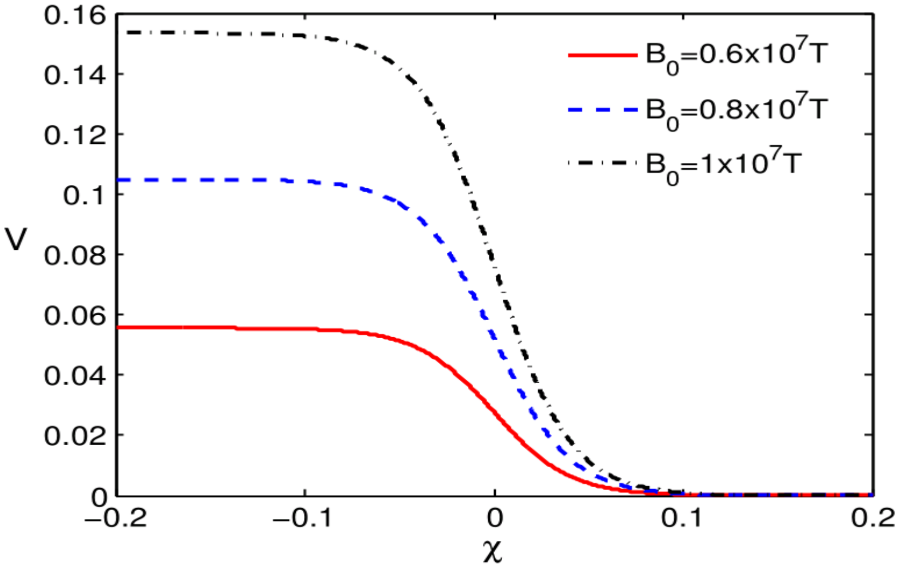

Figure 2: The magnetoacoustic shock waves against χ for different values of B_0 with $n_{e0} = 10^{34}$, $\eta_0 = 0.02$, $p = 0.6$, $\gamma = 0.59$, $\alpha = 0.015$ and $U_0 = 0.5$.

Figure.3 displays the effect of the normalized viscosity coefficient η_0 on the MA shock structures in the absence of dispersion effects (i.e. when the dispersion coefficient $G \to 0$). It is found from this figure that the higher viscosity leads to wider monotonic shocks while its amplitudes are approximately stable. In fact, only normalized widths of the monotonic shock waves are dependent on the normalized viscosity coefficient η_0 via the dissipation coefficient R as shown in shock solution to Burger's equation given by Eq. (61). This suggests that measuring the thickness of shocks in a plasma may be a possible method of determining the viscosity of the plasma.

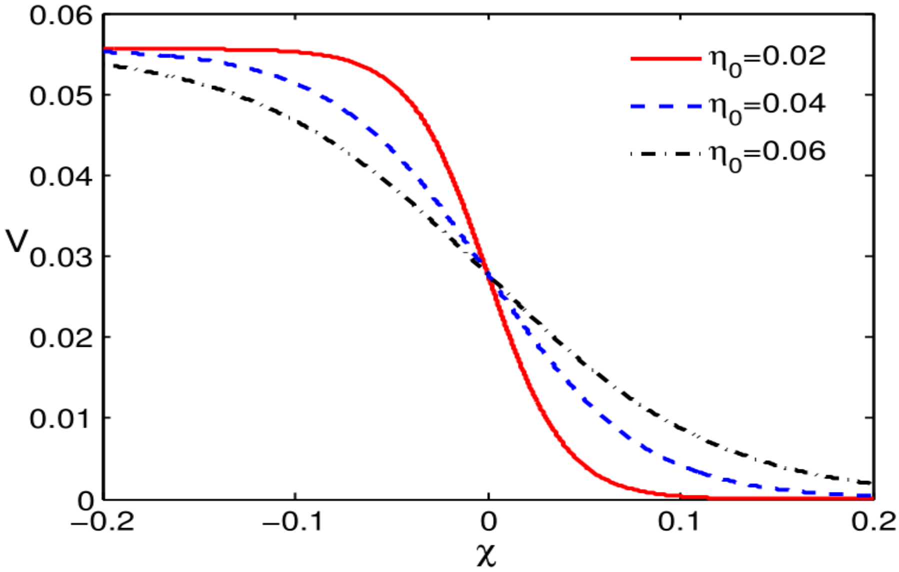

Figure 3: The magnetoacoustic shock waves against χ for different values of $\eta_0$using Eq. (61), with $n_{e0} = 10^{34}$, $B_0 = 0.6 \times 10^7\ T$, $p = 0.3$, $\gamma = 0.59$, $\alpha = 0.015$ and $U_0 = 0.5$.

Figure 4 shows the monotonic shock waves of the MA waves predicted by the two models, namely, with the inclusion of the exchange-correlation effects (solid curve) and without exchange-correlation effects (dashed curve). For the typical quantum plasma parameters we have used, there is a significant difference in both the amplitude and width of the shocks between the two models. It is observed that the exchange-correlation effects lead to a decrease in both the amplitude and the width of monotonic shock waves. It is important to mention here that the exchange-correlation force plays a dominating role of dispersion over the other quantum and pressure gradient forces, where the amplitude and width of the monotonic shock waves are inversely proportional with dispersion coefficient.

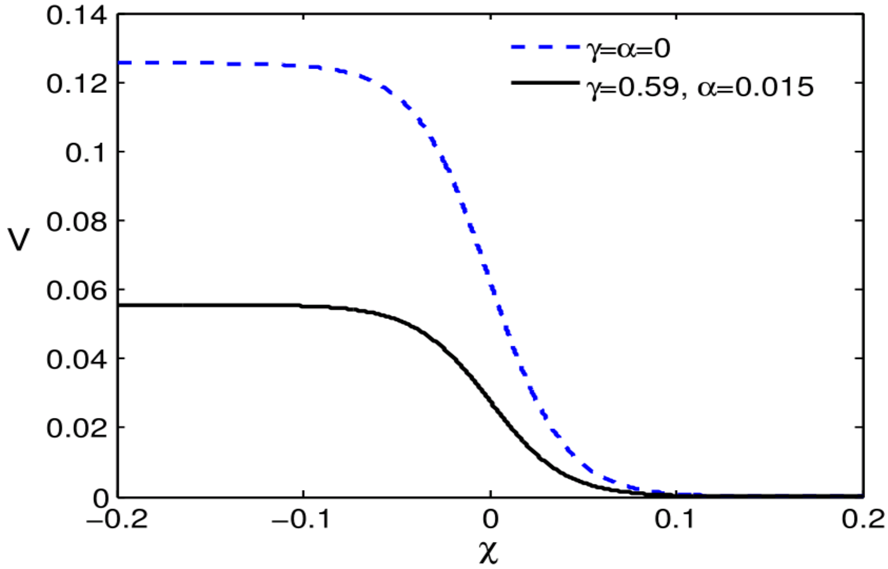

Figure 4: The magnetoacoustic shock waves against χ, with and without exchange-correlation effects, with with $\eta_0 = 0.02$, $n_{e0} = 10^{34}$, $B_0 = 10^6\ T$, $p = 0.6$and $U_0 = 0.5$.

Now, it is interesting to investigate the dependence of the oscillatory shock wave structures of the MA waves on the exchange-correlation effects, positrons concentration p, magnetic field strength B_0, electrons density n_{e0}, and viscosity coefficient η_0. Figure 5 shows the effects of the exchange-correlation on the MA oscillatory shock waves. It is seen from Fig. 5 that in the presence of the exchange-correlation, the amplitude of the oscillatory shock wave increases. This means that the presence of the exchange-correlation in the system reduces the dissipation of energy in the system. Figure 6 shows the effect of the presence of positrons in the system (via the parameter p) on the oscillatory shock profile. It is clear from this figure that the amplitudes of the oscillatory shock waves are found to be enhanced by the increase of p. Furthermore, Fig.7 indicates that the amplitudes of the oscillatory shock waves are enhanced with the density of electrons n_{e0}. The effect of an external magnetic field on the profiles of the oscillatory shock waves is shown in Fig. 8. It is seen from this figure that by increasing the values of magnetic field strength B_0, decreasing the the amplitudes of the oscillatory shock waves.

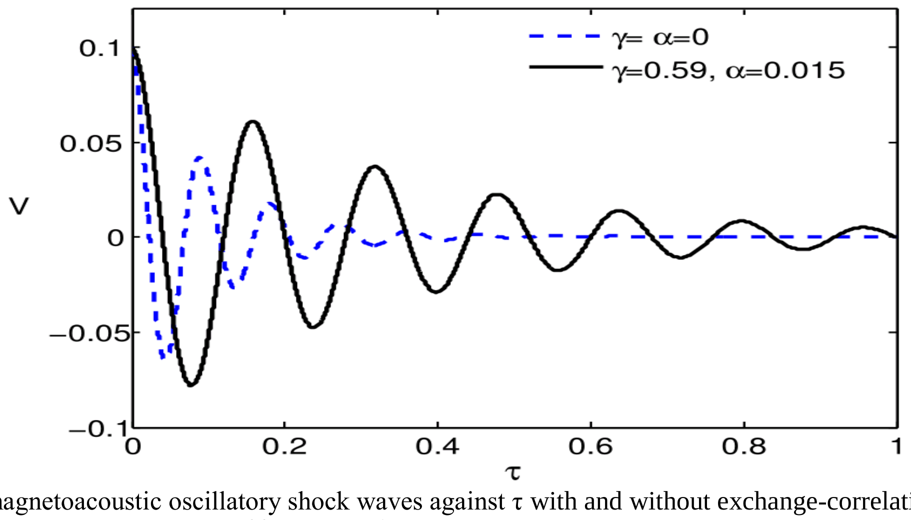

Figure 5: The magnetoacoustic oscillatory shock waves against τ with and without exchange-correlation effects, with $\eta_0 = 0.002$, $n_{e0} = 10^{34}$, $B_0 = 10^6\ T$, $\gamma = 0.59$, $\alpha = 0.015$, $p = 0.3$ and $U_0 = 0.5$.

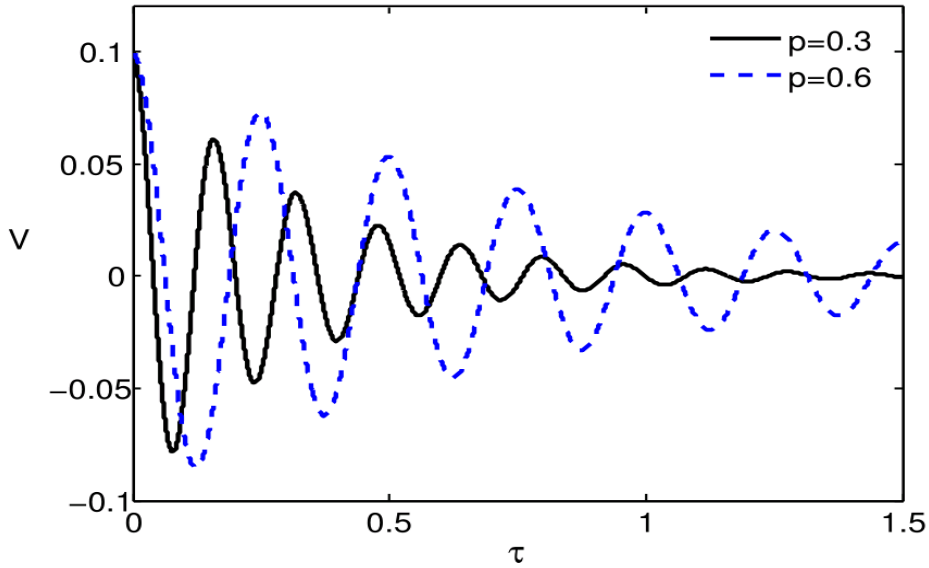

Figure 6: The magnetoacoustic oscillatory shock waves against τ for different values of p with $\eta_0 = 0.002$, $n_{e0} = 10^{34}$, $B_0 = 10^6\ T$, $\gamma = 0.59$, $\alpha = 0.015$ and $U_0 = 0.5$.

Finally, from the Fig. 9, we can see that the viscosity coefficient η_0 would lead to a reduce of the amplitude of the oscillatory shock waves. For low values of η_0, the dissipation of energy is fairly slow leading to a more periodic shock wave. On further increase in the values of the viscosity coefficient, the effect of dissipation gets enhanced in the system, and resulting in lesser periodic shock wave in the system. However, when the dissipative effect is large enough, we have a completely monotonic shock profile without any oscillation.

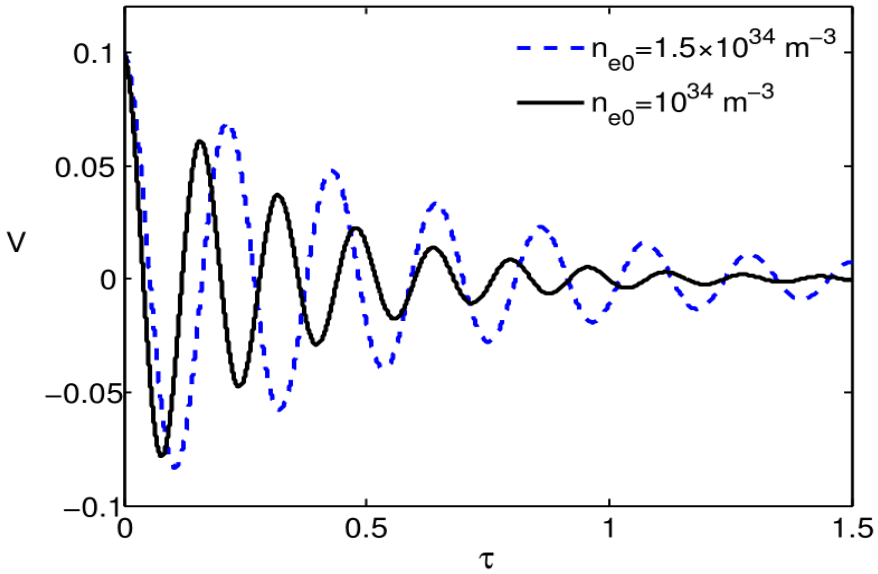

Figure 7: The magnetoacoustic oscillatory shock waves against τ for different values of n_{e0} with $\eta_0 = 0.002$, $p = 0.3$, $B_0 = 10^6\ T$, $\gamma = 0.59$, $\alpha = 0.015$ and $U_0 = 0.5$.

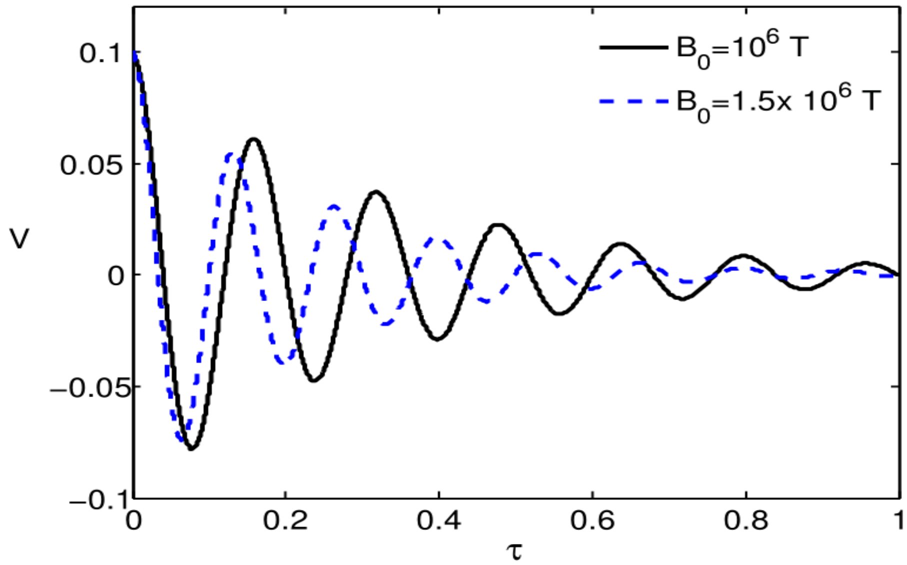

Figure 8: The magnetoacoustic oscillatory shock waves against τ for different values of B_0 with $\eta_0 = 0.002$, $p = 0.3$, $n_{e0} = 10^{34}\ m^{-3}$, $\gamma = 0.59$, $\alpha = 0.015$ and $U_0 = 0.5$.

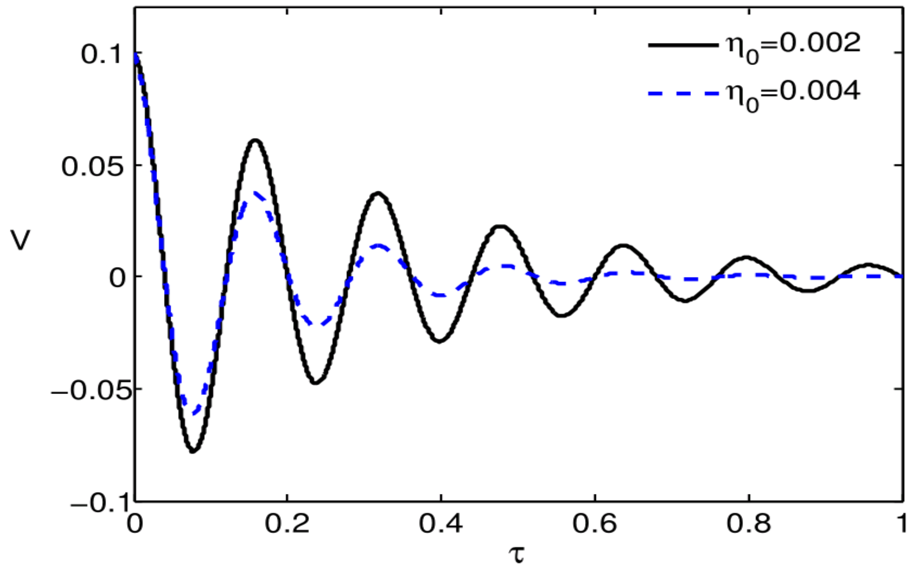

Figure 9: The magnetoacoustic oscillatory shock waves against τ for different values of η_0, with $p = 0.3$, $n_{e0} = 10^{34}\ m^{-3}$, $B_0 = 10^6\ T$, $\gamma = 0.59$, $\alpha = 0.015$ and $U_0 = 0.5$,

6. Conclusion

we have investigated the low frequency MA waves in magnetized quantum electrons-positrons-ions plasmas using the QMHD theory, including the contributions of exchange-correlation in the presence of electrons/positrons spin effects as well as the contribution of ions viscosity in the system. The KdVB equation is derived using the reductive perturbation technique. The analytical solusions of the KdVB are obtained as well. It is found that the quantum magneto-plasma system under consideration supports both the MA solitary wave and the MA shock waves depending on the values of the plasma parameters The necessary condition for the existence of the oscillatory and monotonic shock waves are discussed as well. It is also found that the amplitude and width of the monotonic shock waves increases with increasing the magnetic field strength B_0 and positron concentration (vai the parameter p) while. the presence of exchange-correlation effect leads to a decrease in the both width and amplitude of the monotonic shock waves. Futher, it is found that the monotonic shock waves become wider due to the increase in the viscosity coefficient η_0 whereas increasing the viscosity coefficient would lead to a reduce of the amplitude of the oscillatory shock waves. The results of the current investigation coud have a role in understanding dense magneto-plasma situations such as astrophysical plasma where the quantum mechanical effects of electrons and positrons are included to describe the dense astrophysical plasma systems.

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