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Analysis of laminated Composite Structures

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Abstract

In this paper, we study the distribution of the constraints according to the thickness and analyze the mechanical behavior in inflection, while insisting on the interactions with the interfaces of the layers and the vicinities of the limiting edges. We detail these stages through the application by mentioning the characteristics of each model taken independently of the others. The results are presented in the form of a summary table and figures. To compare the various models of the theory of the plates, this characteristic of the edges enables us to consider the precision of the sinusoidal model compared to the traditional models of the plate theory. We are interested in one of the current problems of the multilayer laminated composites, which is the analysis of the distribution of the stress fields in the vicinities of the edges and in the interfaces of the layers, which play a crucial role in the mechanical resistance of the laminates.

Keywords: laminated structures, composite materials, finite element method.

الملخص: في هذا البحث قمنا بدراسة توزيع القيود حسب سمكها ونحلل السلوك الميكانيكي في الانعطاف مع التركيز على التفاعلات مع واجهات الطبقات ومحيط الحواف المحددة، ونقوم بتفصيل هذه المراحل من خلال ذكر خصائص كل نموذج مأخوذ بشكل مستقل عن النماذج الأخرى. يتم عرض النتائج في شكل جدول موجز واشكال المنحنيات. من أجل مقارنة النماذج المختلفة لنظرية الصفائح نلاحظ أن هذه الخاصية للحواف تمكنا من النظر في دقة النموذج الجيبي مقارنة بالنماذج التقليدية لنظرية الصفائح. نحن مهتمون بواحدة من المشاكل الحالية للمركبات الرقائقيه متعددة الطبقات وهي تحليل توزيع حقول الإجهاد في المناطق المجاورة للحواف وفي واجهات الطبقات التي تلعب دورا أساسيا في المقاومة الميكانيكية للصفائح.

1. Introduction

Nowadays, the further development of composite materials has attracted the attention of scientists and engineers in various fields, such as aerospace, transportation, and other branches of civil and mechanical engineering. In recent years, there has been an increasing number of applications in which composite materials are involved.

It is used in most engineering fields, especially in the aerospace industry, because of its lightweight, high specific strength and stiffness, corrosion resistance, and high thermal resistance. They should be able to maintain their original shape and strength when subjected to thermal and mechanical loads. Thus, the composite material should be able to carry fatigue and creep. A good performance of composite materials depends on good adhesion at the interface between fiber and matrix. [7], [12], [17], [19]. Many researchers have tried to analyze the casting process. Thomas et al. (1987) developed a two-dimensional mathematical model to predict internal stress generated in a steel ingot using the Finite Element Method. [13], [14]. The modeling of the laminated structures is based on the theory

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of the "thick" plates introduced by E Reissner [16] and R. D. Mindlin [18], in the case of the isotropic homogeneous mediums considering that this theory is poorly adapted for the study of the composite plates, of many authors [1],[4], [6], [10], [7], [14] proposed his improvement by giving a more or less refined field of displacements. It is noted that the refined theory considers the warping of the trans versant segment and gives a good approximation for the constraints.
The distribution of stresses shear transversely can take a parabolic or sinusoidal form in the thickness of the plate [6], [9], [10], [21]. A typical composite structure consists of a system of layers bonded together. The layers can be made of different isotropic or anisotropic materials and have different structures, thicknesses, and mechanical properties. In contrast to typical layers whose essential properties are determined experimentally, the laminate characteristics are usually calculated using information concerning the number of layers, their stacking sequence, and geometric and mechanical properties, which should be known. This requires more careful design and analysis for composite structures to be addressed to account for structural behavior accurately. This characteristic does not make it possible to model with precision the interactions that develop with the interfaces of layers and to check the transfer effects of load, which is paramount for the design of laminated [15], [21], [22]. Among the solutions proposed to solve this problem, we quote the work of Pagano [4], [6] and Srinivas [2], [3], [12], [13], which proposed three-dimensional solutions to obtain exact solutions for the behavior of the edges of the laminates, as well as the approach of La devèze [2], [5], [8], [10] which proposed to solve a three-dimensional problem in the zone of edge and a two-dimensional problem inside the plate. The characteristic of these models is to consider the warping of the transverse segment and to check precisely all the limiting conditions at the borders. The finite element method can analyze properties and processes in composite structures. We can be attractive in some instances, mainly when the repetitive mesh of the lattice is complex, to reduce the size of the problem by defining a continuous medium is equivalent to the structure considered. Mainly, we are interested in one of the current problems of the multilayer laminated composites, which is the analysis of the distribution of the stress fields in the vicinities of the edges and in the interfaces of the layers, which exploit a paramount role in the mechanical resistance of the laminates [20], [21. For the forecast of the mechanical behavior of each layer constituting the laminate, in the case of the reinforcement with parallel fibers.

2. Laws of behavior

We place within the framework of the theory of the plates. Consequently, by introducing an unspecified function q and by using a formulation of the problems in extreme cases in such a way as to satisfy the principle of the virtual powers and all the limiting conditions of the borders, we lead to the relations of the generalized efforts, which are written in the form:

$$N_{ij} = A_{ijkh}\frac{\partial v_k}{\partial x_h} - D_{ijkh}\frac{\partial^2 w}{\partial x_k \partial x_h} + \tilde{D}_{ijkh}\frac{\partial \gamma_k}{\partial x_h} \quad (1.1)$$

$$M_{ij} = D_{ijkh}\frac{\partial v_k}{\partial x_h} - B_{ijkh}\frac{\partial^2 w}{\partial x_k \partial x_h} + b_{ijkh}\frac{\partial \gamma_k}{\partial x_h} \quad (1.2)$$

$$\tilde{M}_{ij} = \tilde{D}_{ijkh}\frac{\partial v_k}{\partial x_h} - b_{ijkh}\frac{\partial w}{\partial x_k \partial x_h} + \tilde{B}_{ijkh}\frac{\partial \gamma_k}{\partial x_h} \quad (1.3)$$

$$\tilde{Q}_i = \tilde{A}_{i3j3}\gamma_j \quad (1.4)$$

i, j, k, h = 1, 2)

3. Applications

3.1 Case of a plate laminated in simple supports on two edges

Given the intricate nature of the problem formulation, our focus is on a simplified case study involving laminated, orthotropic plates subjected to transverse loads, which can be concentrated or distributed [2], [7]. This presentation applies to all plate models [4], [6], [11], demonstrating the various calculation stages necessary to obtain equilibrium equations and ensure their resolution by defining boundary conditions and adapting model assumptions to our calculations. Specifically, we delve into the distribution of constraints across the thickness and scrutinize the mechanical behavior in inflection, emphasizing the interactions with the layer interfaces and the edges.

The translation of these boundary conditions depends on the nature of the conditions cinematics and imposed mechanics in terms of displacements is done by taking account of the law of behavior, given by the relations (1)

3.2. Equilibrium equations and boundary conditions

It is about a plate laminated, orthotropic, and made up of four layers, directed with ($90^0/0^0$, $/0^0/90^0$) and subjected to a transverse load distributed $q(x_1, x_2)$. It is supposed that the plate is in simple supports on the two parallel edges (fig.1). The mechanical characteristics are identical to those used by Srinivas [2] and Ladevèze [10]. By using the same semi-infinite formulation of the plate and while being limited to the static study, the equilibrium equations are written in their following general form:

$$-B_{1111}\frac{\partial^4 w}{\partial x_1^4} - 2(B_{1122}+2B_{1212})\frac{\partial^4 w}{\partial x_1^2\partial x_2^2} - B_{2222}\frac{\partial^4 w}{\partial x_2^4}$$

$$+b_{1111}\frac{\partial^3\gamma_1}{\partial x_1^3} - (b_{1122}+2b_{1212})\left(\frac{\partial^3\gamma_1}{\partial x_1\partial x_2^2}+\frac{\partial^3\gamma_2}{\partial x_2\partial x_1^2}\right) + b_{2222}\frac{\partial^3\gamma_2}{\partial x_2^3} + q(x_1, x_2) = 0 \qquad (2.1)$$

$$-b_{1111}\frac{\partial^3 w}{\partial x_1^3} - (b_{1122}+2b_{1212})\frac{\partial^3 w}{\partial x_1\partial x_2^2} +$$

$$\tilde{B}_{1111}\frac{\partial^2\gamma_1}{\partial x_1^2} + (\tilde{B}_{1122}+2\tilde{B}_{1212})\frac{\partial^3\gamma_2}{\partial x_2\partial x_1} + \tilde{B}_{1212}\frac{\partial^2\gamma_1}{\partial x_2^2} - \tilde{A}_{1313}\gamma_1 = 0 \qquad (2.2)$$

$$-b_{2222}\frac{\partial y}{\partial x}(b_{1122}+2b_{1212})\frac{\partial^3 w}{\partial x_2\partial x_1} + \breve{B}_{2222}\frac{\partial^2\gamma_2}{\partial x_2} + \left(\breve{B}_{1122}+\breve{B}_{1212}\right)\frac{\partial^2\gamma_1}{\partial x_1\partial x_2} + \breve{B}_{1212}\frac{\partial^2\gamma_2}{\partial x_1} - \breve{A}_{2222}\gamma_{2=0} \qquad (2.3)$$

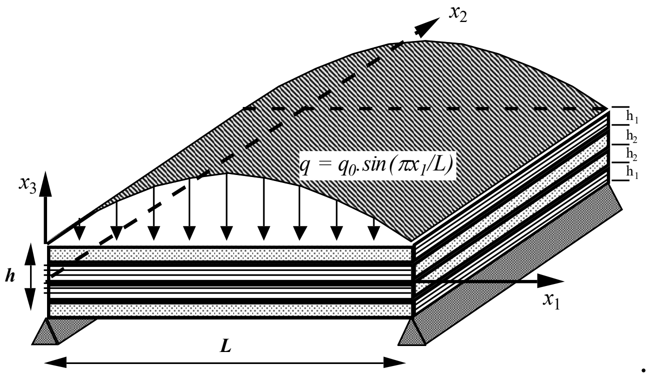

Fig.1. Geometry of the laminated plate and boundary conditions
(L = 1 m, H_1 = 0,01m, H_2 = 0,04 m, Q_0 = -1 MPa)

To solve this system, we introduce the boundary conditions (cinematics and natural) on the edges. Thus, by considering the case of (Fig.1), where the plate is in simple supports on the axis $x_1 = 0$

$x_1 = L$ and free on the two others, the boundary conditions are written in terms of displacements by taking account of the laws of behavior (1) :

For the natural conditions on the free edges (x_2 = 0 and x_2 = L, whatever x_1)

$$-B_{1122}\frac{\partial^3 w}{\partial x_1^2 \partial x_2}-B_{2222}\frac{\partial^3 w}{\partial x_2^3}-2B_{1212}\frac{\partial^4 w}{\partial x_1^2 \partial x_2}$$
$$+b_{1122}\frac{\partial^2 \gamma_1}{\partial x_1 \partial x_2}+b_{1212}\left(\frac{\partial^2 \gamma_1}{\partial x_1 \partial x_2}+\frac{\partial^2 \gamma_2}{\partial x_1^2}\right)+b_{2222}\frac{\partial^2 \gamma_2}{\partial x_2^2}=0 \qquad (3.1)$$

$$-B_{1122}\frac{\partial^2 w}{\partial x_1^2}-B_{2222}\frac{\partial^2 w}{\partial x_2^2}+b_{1122}\frac{\partial \gamma_1}{\partial x_1}+b_{2222}\frac{\partial \gamma_2}{\partial x_2}=0 \qquad (3.2)$$

$$\tilde{B}_{1212}\left(\frac{\partial \gamma_1}{\partial x_2}+\frac{\partial \gamma_2}{\partial x_1}\right)-2B_{1212}\frac{\partial^2 w}{\partial x_1 \partial x_2}=0 \qquad (3.3)$$

For the conditions cinematics on the superficial edges of supports (x_1 = 0 and x_1 = L, whatever x_2):

$$w=\gamma_1=\gamma_2=0 \qquad (3.4)$$

It is pointed out that these equations include the general case of the theory of the plates where the function $g(x_3)$ is any type of model. Thus, within the framework of the theory of Reisner and the refined theories (Reddy, Sinusoidal), we keep the same equilibrium equations (2), and we modify the relations that govern the coefficients of elasticity by specifying the form of the function $g(x_3)$ according to the type of the model considered. Within the framework of the theory of Kirchhoff where ($g(x_3)$= 0) and of which we neglect the effects of transverse shearing ($\gamma_1 = \gamma_2 = 0$), the equilibrium equations are summarized:

$$-B_{1111}\frac{\partial^4 w}{\partial x_1^4}-2(B_{1122}+2B_{1212})\frac{\partial^4 w}{\partial x_1^2 \partial x_2^2}-B_{2222}\frac{\partial^4 w}{\partial x_2^4}+q(x_1,x_2)=0 \qquad (4)$$

4. Resolution of the problem

The resolution of the problem consists in the forms of developments in the Fourier series according to the technique of Naiver relating to the cases of the simple bearing plates on all the borders of the plate or that of Levy, which treats the case of the simple bearing plates on two parallel edges and letting at the other edges the free possibility, be supported or embedded. In the general case of a rectangular plate, subjected to conditions cinematics on the four borders, displacements and the transverse load $q(x_1 , x_2)$ are developed according to the double Fourier series.

$$q(x_1,x_2)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \mathrm{q}_{\mathrm{mn}}\, sin\frac{n\pi x_1}{L}\, sin\frac{n\pi x_2}{l} \qquad (5.1)$$

$$u_{\mathrm{i}}(x_1,x_2)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \mathrm{U}^{\mathrm{i}}_{\mathrm{mn}}\, sin\frac{n\pi x_1}{L}\, sin\frac{n\pi x_2}{l} \qquad (5.2)$$

However, we limit ourselves in our resolution to the cases of the simple bearing plates on two parallel edges (fig.1), which can be supported or embedded on the two other free edges, as in the technique of Levy. Thus, by proposing a uniform transversal loading according to the direction x_2 and the equilibrium equations, displacements and the transverse load are put in the form.

$$w(x_1,x_2)=W(x_2)\,sin(\pi x_1 /h) \qquad (6.1)$$

$$\gamma_1(x_1, x_2) = \Gamma_1(x_2)\cos(\pi x_1 / h) \tag{6.2}$$

$$\gamma_2(x_1, x_2) = \Gamma_2(x_2)\sin(\pi x_1 / h) \tag{6.3}$$

$$q(x_1) = q_0 \sin(\pi x_1 / h) \tag{6.4}$$

Work thus consists of supplementing the solutions (6) by determining the functions $W(x_2)$ $\Gamma_i(x_2)$ so that they check the equilibrium equations and the boundary condition on the edges, with the added restriction of imposing a cinematics condition of simple supports on two parallel edges, as required by the technique of Levy. We note that this proposed form of displacements satisfies well the boundary conditions of simple supports on the axis $x_1 = 0$ and $x_1 = L$ whatever x_2 :

$$w(0, x_2) = w(L, x_2) = 0 \tag{7.1}$$

$$\gamma_1(0, x_2) = \gamma_2(L, x_2) = 0 \tag{7.2}$$

$$M_{11}(0, x_2) = M_{11}(L, x_2) = 0 \tag{7.3}$$

$$\tilde{M}_{11}(0, x_2) = \tilde{M}_{11}(L, x_2) = 0 \tag{7.4}$$

Thus, by introducing the relations of displacements into the equilibrium equations (2), we obtain the system of equations:

$$\begin{aligned}&-a^4 B_{1111} W + 2a^2 (B_{1122} + 2B_{1212}) \frac{\partial^2 W}{\partial x_2^2} - B_{2222} \frac{\partial^4 W}{\partial x_2^4} \\ &+ a^3 b_{1111} \Gamma_1 - (b_{1122} + 2b_{1212})(a \frac{\partial^2 \Gamma_1}{\partial x_2^2} + a^2 \frac{\partial \Gamma_2}{\partial x_2}) + b_{2222} \frac{\partial^3 \Gamma_2}{\partial x_2^3} + q_0 = 0\end{aligned} \tag{8.1}$$

$$\begin{aligned}&-a^3 b_{1111} W - a(b_{1122} + 2b_{1212}) \frac{\partial^2 W}{\partial x_2^2} \\ &+ a^2 \tilde{B}_{1111} \Gamma_1 + a(\tilde{B}_{1122} + 2\tilde{B}_{1212}) \frac{\partial \Gamma_2}{\partial x_2} + \tilde{B}_{1212} \frac{\partial^2 \Gamma_1}{\partial x_2^2} - \tilde{A}_{1313} \Gamma_1 = 0\end{aligned} \tag{8.2}$$

$$\begin{aligned}&-b_{2222} \frac{\partial^3 W}{\partial x_2^3} - a^2 (b_{1122} + 2b_{1212}) \frac{\partial W}{\partial x_2} \\ &+ \tilde{B}_{2222} \frac{\partial^2 \Gamma_2}{\partial x_2^2} - a(\tilde{B}_{1122} + 2\tilde{B}_{1212}) \frac{\partial \Gamma_1}{\partial x_2} - (a^2 \tilde{B}_{1212} + \tilde{A}_{2323}) \Gamma_2 = 0\end{aligned} \tag{8.3}$$

With: $a = \frac{\pi}{l}$

The general solution of this system, without a second member (q_0 = 0), is given by:

$$W(x_2) = A_i \exp(\beta_i x_2) \tag{9.1}$$

$$\Gamma_1(x_2) = B_i \exp(\beta_i x_2) \tag{9.2}$$

$$\Gamma_2(x_2) = C_i \exp(\beta_i x_2) \tag{9.3}$$

Where A_i B_i C_i are the constants to determine by using the boundary conditions natural and cinematics and β_i are the roots of the equations characteristic associated with the system of differential equations:

$$\begin{bmatrix} a_{11}(\beta_i) & a_{12}(\beta_i) & a_{13}(\beta_i) \\ a_{21}(\beta_i) & a_{22}(\beta_i) & a_{23}(\beta_i) \\ a_{31}(\beta_i) & a_{32}(\beta_i) & a_{33}(\beta_i) \end{bmatrix} \begin{Bmatrix} A_i \\ B_i \\ C_i \end{Bmatrix} = \left[A_{ij}(\beta_i)\right] \begin{Bmatrix} A_i \\ B_i \\ C_i \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{10}$$

The coefficients β_i are obtained while canceling to determine the matrix A_{ij} :

$$\det\lfloor A_{ij}(\beta_i)\rfloor = 0 \quad (11)$$

Finally, the general solution of the equilibrium equations is obtained by superimposing the particular solution and the homogeneous solution without a second member (q_0 = 0) and by taking account of the cases where the roots β_i are simple or multiple. The form of the general solution is written

$$w(x_1,x_2) = \left(\sum_{i=1}^{p} A_i exp(\beta_i x_2) + \sum_{i=p+1}^{n} (A_i.x_2 + A_{i+1}) exp(\beta_i x_2) + w_\beta\right) sin\frac{\pi x_1}{L} \quad (12.1)$$

$$\gamma_1(x_1,x_2) = \left(\sum_{i=1}^{p} B_i exp(\beta_i x_2) + \sum_{i=p+1}^{n} (B_i.x_2 + B_{i+1}) exp(\beta_i x_2) + \gamma_{1\beta}\right) cos\frac{\pi x_1}{L} \quad (12.2)$$

$$\gamma_2(x_1,x_2) = \left(\sum_{i=1}^{p} C_i exp(\beta_i x_2) + \sum_{i=p+1}^{n} (C_i.x_2 + C_{i+1}) exp(\beta_i x_2) + \gamma_{2\beta}\right) sin\frac{\pi x_1}{L} \quad (12.3)$$

Where p is the number of the simple roots and $n-p$ is the number of the multiple roots.

5. Results

The results are presented in the form of a summary table and figures. In order to compare the various models of the theory of the plates, we keep as a reference the results resulting from calculations by finite elements method within the framework of elasticity three-dimensional Table I represents the maximum values of the arrow and the constraints dimensionless by using the following relations:

$$\bar{\sigma}_{ij}(x_1,x_2,x_3) = \frac{\sigma_{ij}(x_1,x_2,x_3)}{q_0} \quad (13.1) \qquad \bar{w}(x_1,x_2) = \frac{100.C_{2222}}{hq_0} w(x_1,x_2) \quad (13.2)$$

Table. I. *Maximum values of the dimensions constraints and displacement.*

	$\bar{w}$ (L/2,L/2)	$\bar{\sigma}_{11}$ (L/2,L/2,h/2)	$\bar{\sigma}_{22}$ (L/2,L/2,h/2)	$\bar{\sigma}_{12}$ (3L/4,3L/2,h/2)	$\bar{\sigma}_{13}$ (L/4,L/2,0)	$\bar{\sigma}_{23}$ (L/2,L/4,0)
Kirchhoff Love	83.78	124.22	20.72	3.25		
Reissner	117.51	128.21	17.51	5.35	2.11	0.21
Reddy	112.31	119.15	18.91	5.33	2.28	2.24
Sinus	112.73	123.23	18.52	5.41	2.41	0.26
E.F.	116.42	130.13	18.47	5.39	2.39	0.28

In the representative curves, we limited the comparisons of three significant models. The results obtained by the sinusoïdal model, compared with the traditional model of *Reissner* and numerical calculations by finite elements method. We note that calculations by finite elements method required using a three-dimensional mesh consisting of 2770 cubic elements with 20 nodes (Fig. 2). The figure (3 to 8) compares the results from the abovementioned models. On the other hand, we note an excellent agreement of the results relating to the evolution arrow along the axis (x_1, x_2 = L/2, x_3=0)(Fig. 3). In addition, in the figures relating to the distribution of the constraints according to the thickness of the plate, in the vicinity of the free edge (fig.4), we note the inaccuracy of the model of *Reissner* for the constraints σ_{13} and σ_{23}. The errors are estimated at 1,7 % for the arrow and 1,9 % for the constraint σ_{11}. The figures relating to the constraints (σ_{11} , σ_{22} , σ_{12} , σ_{13} and σ_{23} (Fig. 4 to 8)) confirm the excellent behavior of the sinusoidal model compared to three-dimensional calculations by the finite elements method. However, we note the significance of the discontinuity of the constraints to the interfaces of the layers of the plate. The differences between the constraints in the vicinities of the interfaces decrease for the model sinusoïdal, compared with the model of Reissner. The distribution of the constraints in the two external layers highlighted effects edges. We note that this characteristic of the edges makes it possible to consider the precision of the model sinusoidal compared to the traditional models of the theory of the plates.

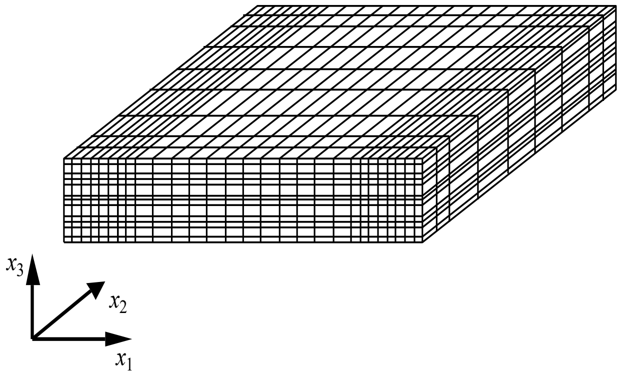

Fig. 2. *Grid of the laminated plate*

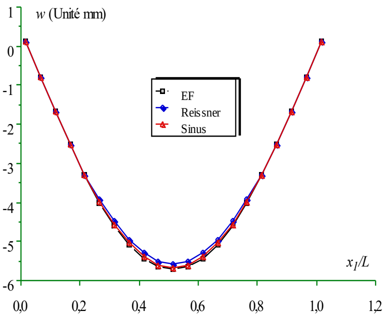

Fig. 3.*The arrow W in average plan along the axis (X $_1$, L/2, 0).*

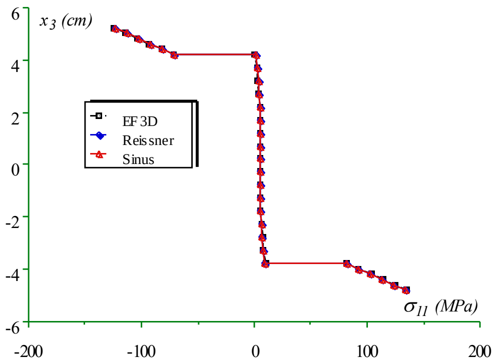

Fig. 4.*Distribution of the constraint* σ_{11} *(L/2, L-h, 0) according to the thickness.*

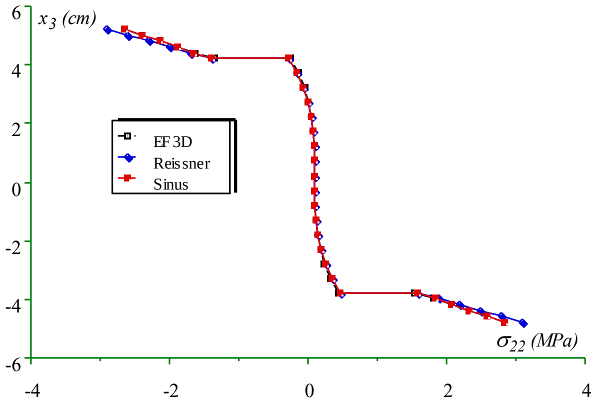

Fig. 5.*Distribution of the constraint* σ_{22} *(L/2, L-h, 0) according to the thickness.*

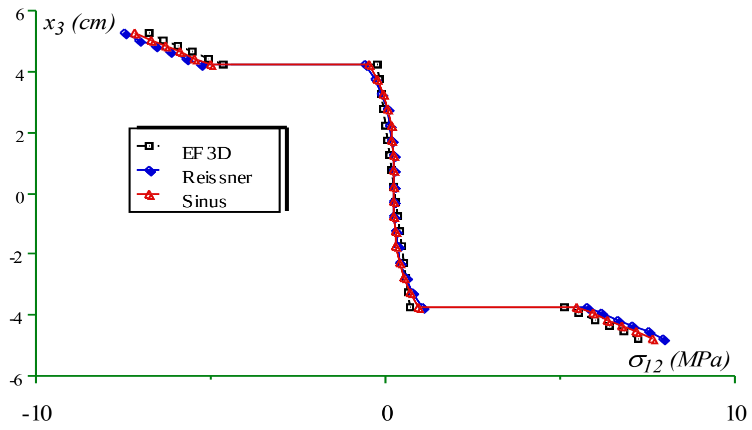

Fig.6. *Distribution of the constraint* σ_{12} *(L/2, L-h, 0) according to the thickness.*

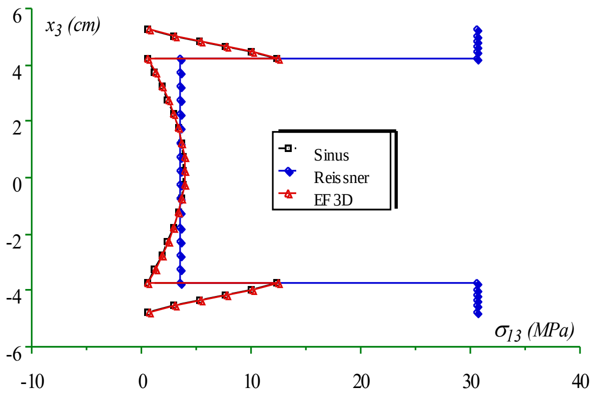

Fig. 7.*Distribution of the constraint* σ_{13} *(L/2, L-h, 0) according to the thickness.*

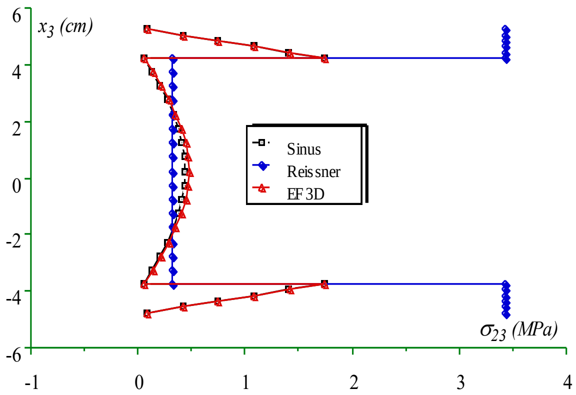

Fig. 8. Distribution of the constraint σ_{23} (L/2, L-h, 0) according to the thickness.

6. Conclusion

In this work, the results obtained are validated starting from the three-dimensional solution and compared with the other ideal models. These results are validated by comparing the arrow values and constraints from the numerical calculation using the finite elements method for plane strain to those obtained from different plates theory. We noted excellent agreement of the results relating to the evolution arrow along the axis, and the distribution of the constraints according to the thickness made it possible to note the discontinuity of the constraints, which can explain the importance of the interfaces in the performances of the laminated structures.

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